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A Heterogeneous Fleet Vehicle Waste Collection Problem with Various Zones and Intermediate Facilities

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Abstract

Waste collection is one of the important problems that can affect urban life. A multi-zone waste collection routing problem is proposed in this paper considering intermediate facilities, depot centers and bins as network's nodes. In this model heterogeneous fleet vehicles are considered to load wastes from bins from every zone and then unload them in disposal centers which belong to that zone and each zone can include just one depot center. A mixed integer programming model is considered to solve the problem with considering new assumptions. The objective function attempt to minimize the total operation cost of vehicles including fixed and variable costs and also minimizes the number of vehicles are used and last term of objective function is the absolute difference between the total number of unloading at each disposal center and the average number of unloading at all of the disposal centers. Finally in order to show applicability of proposed model, two small large numerical examples are presented.

Keywords: Heterogeneous fleet vehicles

Introduction

The collection of urban solid waste is one of the most troublesome operational problems faced by local authorities in every city. Recently, according to the concerns about health, cost, and environment, many municipalities, especially in developed countries, have been forced to manage solid waste and examine its cost – efficiency and environmental effects. During the past 20 years, there have been numerous technological progresses and new developments in the waste industry [15].

Detailed data from different case studies represent that apply a collection system from one community to another cannot be applied. Refuse attribute depends on numerous factors like food habit, cultural traditions, climates and socio-economic conditions. The suitable method for waste collection depends on the characteristic and composition of the waste.

Waste collection is a type of vehicle routing problem that is applied in real word, widely. There are two extensive categories of models in the waste collection problems, fundamentally. The first type is arc routing problem (ARP) and the second is node routing problem. Waste collection problems are usually modeled as Arc Routing Problems that the objective is to find a cost – effective traversal of defined arcs of graph, subject to some other constraints. The demand occurs at the node of the

determined graph. Arc Routing Problem is a suitable way to model waste collection problems in cases most of the bins along a given street section must be collected at the same time and most of the street must be passed.

With node routing it is possible to design a more detailed model, because of considering each bin separately to provide a more detailed solution. Waste management companies have detailed data; therefore node routing is more suitable [10]. In this study the node routing problem is considered, instead of ARP. In waste collection problems the wastes that load by vehicles, unload at special sites such as disposal centers, called intermediate facilities (IF). Intermediate facilities are usually different from depots that vehicles states there. It is considered there are some disposal centers to unload vehicles in this study, so vehicle routing problem with intermediate facilities (VRP-IF) is an appropriate model [10]. VRP-IF extends the VRP that there is a batch of Intermediate Facilities (IF) where vehicles unload the waste and then come back to the depot.

In the literature another class of ARP is considered that is capacitated arc routing problem (CARP), in case of this problem each arc has a nonnegative demand and the aim is to determine a minimum cost traversal of all arcs by the vehicles according to not exceeding from the vehicles

capacities. Each pair of nodes that are connected together constructs an arc that must be serviced by one vehicle exactly. The Capacitated Arc Routing Problem with Intermediate Facilities (CARPIF) was introduced by Ghiani et al. (2004) [9]. The CARPIF is a CARP that considered intermediate facilities and the objective is to find a least-cost tour with all arcs that total vehicle load does not exceed its capacity. In fact, the vehicle departs from the depot to visit all edges and collect demands and unload them at the intermediate facilities, but the cumulative load on each tour should not exceed from the vehicle capacity. The capacitated arc routing problem with intermediate facilities and tour length restrictions (CLARPIF) represented by Ghiani et al. (2004) [9]. The CARP is NP-hard even when it is assumed a single vehicle that is called Rural Postman Problem (RPP) and it is very similar to the VRP [17].

In this research, a fleet of heterogeneous vehicles with restricted capacity is considered using a depot in each zone. All of the vehicles stationed at depots and should depart from depots to collect garbage from bins. It is assumed some disposal centers and numerous bins in each zone. Vehicles can load wastes from bins are stated at another zones but must unload at the disposal centers located at their zones. The aim is to find cost-effective routes to collect waste from bins that are located in different sides of a city. The total cost consists of fixed cost (maintenance, driver) and variable cost (fuel cost) regarding balancing unloads at disposal centers should be minimized in this model. On the other hand, minimization the sum of absolute difference between the length of each trip and the average length of all trips is considered.

Reminder of the paper is organized as follows: in section 2, body of literature in waste collection routing problems is presented. Problem definition, parameters and formulation are presented in section 3. Experimental results from two test problems in order to show applicability of proposed model is presented in section 4 and in the last section, conclusion and future research directions are presented.

Literature Review

The waste collection problem is very similar to vehicle routing problem (VRP). In the classical VRP there are numerous vehicles that are state at depot, each of which start from the depot visit some nodes and return to the depot to determine a cost-effective route. In the literature some papers considered wastes site construction location. This kind of problems is known as the roll-on roll-off problem. Benjamin and Beasley (2010) investigated

such a problem. They supposed a problem with single depot, a batch of waste disposal facilities, a set of customers and an unrestricted number of vehicles that there was a time window associated with opening time of depot for each customer. In their problem, a neighbor set was defined with compatible time window. Two Metaheuristic algorithms by Tabu Search (TS) and also a variable neighborhood tabu search algorithm presented as solution methodologies. Variable indicators proposed to evaluate the efficiency of urban solid waste collection in Kuwait. Computational outcomes displayed that variable neighborhood search was more efficient than other Metaheuristic in cases alike [5]. Alumur and Kara (2007) proposed a multi-objective location-routing model to transport hazardous wastes that have long-time risks to environment in case of turkey. Some of industries produce hazardous wastes and there are hazardous household wastes like batteries, antifreeze, gasoline, household cleaning products, pesticides, thinners and oil-based paints. They attempted to location treatment and disposal centers and specify technology of them and routes to disposal centers so that minimize total cost and transportation risks [1].

The vehicle routing scheduling problem (VRSP) is a VRP that includes time constraints such as total time limitation, time window and priorities for the service (pick-up and delivery). The network model is suitable for such a problem. Tung and Pinnoi (2000) represented a heuristic approach to solve a waste collection problem in Vietnam. They proposed a mixed integer programming model for a VRSP considering several time windows for customers and also inter-arrival time limitations. They presented a heuristic procedure including two parts which first part that was taken from Solomon (1987), constructs a route and the second one improves route which obtain in the first part [8]. Amponsah and Salhi (2004) developed a useful look-ahead heuristic method for collection of trashes. A fleet of vehicles whose capacity cannot be exceeded and several dump sites to unloading vehicles are existed. They used a look-ahead approach based on a bi-objective model to minimization of cost and environmental effects and showed that their heuristic strategy is efficient. The capacitated Arc Routing Problem (CARP) in real problems cannot be solved by exact methods; therefore heuristics are used in these cases. One approach is route first-cluster second where first find a giant tour, afterward decompose it into a batch of routes which are feasible according to the vehicle capacity. This approach was used by Boltrami and Bodin (1974) [2]. Ong et al. (1990) represented a computerized vehicle routing system in Singapore. It is required considerable amount of time on data

collection to solve a VRSP. The objective was traveled distance by vehicles minimization to save fuels and vehicle maintenance work and reduce operation costs. They utilized the route first–cluster second approach to solve this multiple-vehicle routing problem [16]. Mourão (2000) presented a similar approach for solving household refuse collection problem with lower-bounding method, due to the NP-hardness of CARP. He represented two lower-bounding methods that are based on transportation model and also a three-phase heuristic to obtain near-optimal solution from the solution generated from the first lower-bounding method [14].

The Stochastic Vehicle Routing Problem (SVRP) can be defined as the VRP that some of the elements such as travel time; number of customers and etc. are stochastic. Nurtio and Kytöjoki (2006) planned vehicle routes for the collection of urban solid waste in two regions in Eastern Finland. Separate containers considered for each type of solid waste was collected separately. Each vehicle only could collect one type of waste. Number and capacity of vehicles are limited and two travels could operate each day and every travel could perform with separate vehicle. The disposal sites have given a time window. The amount of municipal solid waste is available; therefore the average accumulation rate of waste in each container type was estimated. Also, The historical data used to estimating the time window to unload at the disposal site and the travel time. The objective was total distance traveled minimization. This problem is in class of Stochastic Periodic Vehicle Routing Problem with Time Window (SPVRPTW) and a limited number of vehicles. In the literature, optimization of the number of vehicle fleet is almost always a goal [15]. But Nurtio et al. (2006) supposed that the size of vehicle is fixed [15]. Kytöjoki et al. (2007) developed Guided Variable Neighborhood Search (GVNS) heuristic and presented a real-life application in large-scale practical routing problem [12].

The Periodic Vehicle Routing Problem (PVRP) arises when problem is defining routes for all days given T-day period. In the Vehicle Routing Problem with Time Window (VRPTW) the time window is assumed for each customer. Teixeira et al. (2004) planned vehicle routes for the collection of variable kinds of municipal recyclable wastes using heuristic approaches. In their problem, it is assumed two depots, vehicle start and end routes in this two depots and the objective is minimization of distance that vehicles traverse. They utilized PVRP model, because every collection site needs a specific number of collections for each type of waste during the period. The goal was to define the geographic collection zones, vehicle routes and waste type to be

collected on each day in real-life, hence heuristic such as cluster first-route second approach utilized. Their contributions were design a plan to collection three type of waste separately in a long period of time [18]. Bommisetty et al. (1998) considered collection recyclable materials in a large university campus which had numerous buildings. It is assumed a single-day period, therefore the appropriate model for this problem was PVRP. A two phase heuristic algorithm was proposed; because of complexity of this problem that first phase specified that recyclable of which building should be collected on which day of week and second one determined a set of routes each day for the assigned buildings which minimizes whole travel and collection time [6]. Baptista et al. (2002) studied the collection of recycling paper containers. They utilized a PVRP and the aim was to determine when will be each customer visited (considering time horizon). They mentioned that the daily routes cannot be considered separately of allocation of collection/delivery days to customers. The most important feature of this problem was that volume of paper to be collected at each point was random variable. A heuristic method proposed as a solution method. In this method, First initial sequence and visit day combinations were assigned and then an interchange procedure applied to find a better visit day combinations [4].

Aringhiera et al. (2004) considered a pickup and delivery vehicle routing problem, with unit vehicle capacity. The problem was designing a graph model for the collection and disposal of bulky recyclable waste in Italy. In their problem, containers which used to collect different waste materials were of different types, afterward wastes must be picked up to be emptied at appropriate disposal plants and replaced by empty containers alike, Disposal plants were related to the material and located in different sites. Different containers for each type of collected waste materials such as metals, paper, wood, glass garden waste and etc. was assumed and users brought their waste and disposed it into the suitable container, according to the material. If a container was full, a disposal request sent out, consisting of the following two operations, to be performed not unavoidably in this order: first, the full container is brought to a suitable disposal plants to be omitted and second, another empty container alike is brought to the collection point. A fleet of homogenous vehicle was existed at a single depot or at some other sites. The goal was to determine the vehicle route starting and ending at the depot, while minimizing the number of vehicle and the total traveled time. This problem is a special Asymmetric vehicle routing problem (AVRP). It is provided a graph model for this problem, and used heuristic because of problem

solution complexity [3].Crevier et al. (2007), proposed a multi-depot vehicle routing problem with inter-depot routes (MDVRPI). They also presented a tabu search algorithm, which were then combined by a set partitioning approach as a solution method [7]. By considering mentioned previous researches in waste collection routing problem, novelties in proposed model in this paper can be presented as follows:

- Heterogeneous fleet vehicle considering with different capacity, speed and cost, instead of homogenous ones.
- Disposal centers considering as intermediate facilities.
- Permitting the vehicle that cover different zones' bins.

Problem Description

In this research, a multi-zone, heterogeneous fleet of vehicles, scheduling waste collection problem is proposed. The city is divided to several zones and there is single-depot, some disposal centers (a place for disposal waste) and numerous bins in each zone. The waste volume exists in each bin is known. It is assumed there are heterogeneous fleets of vehicle that are available and located in depots and vehicles have capacity constraint and it depends on the type of vehicle. The vehicle departs from depot to collect waste from the bins all over the city so that not exceeding from the vehicles capacity. Hence, it departs to a disposal center to unload waste and starts another trip to collect waste from the other bins that are not empty. Each vehicle from each zone can load wastes from the bins throughout the city, but it must unload in the disposal centers which belong to that zone. Vehicles work until there is no waste in all of the bins. Each vehicle must move to a disposal center to unload at the end of the last trip, hence the rout of this vehicle is finished. Depots have not capacity constraint and there is no limitation for deliveries to disposal facilities.

Considered problem is shown in figure (1), schematically. As can be seen, zones of the city, disposal centers, depots, bins and some of routs are represented. As can be seen in the figure, vehicles from each zone can load wastes from all of the bins in all over the city but they forced to come back to zone which belongs to.

Considered assumptions are represented as follow:

- The city is divided to several zones and there is a depot, some disposal centers and numerous bins in each zone.
- The distance between each pair of nodes is defined and known and the way between

each pair of the points is specific and bidirectional.

- The waste volume of each bin is deterministic and known.
- There are k type of vehicles, on the other hand heterogeneous fleet of vehicles are available. Different type of vehicle has different capacity, speed and cost.
- The operation cost of the vehicle consists of variable cost (fuel cost) and fixed costs (driver salary and maintenance cost); Therefore the operational costs depends on the type of the vehicle, because fuel consumption of different kind of machines is not the same.
- The cost of vehicle in loading and unloading time is same as moving time, because the engine is working in the both situations and it consumes the fuel.

Loading and unloading time at each node depends on the type of vehicle and waste volume.

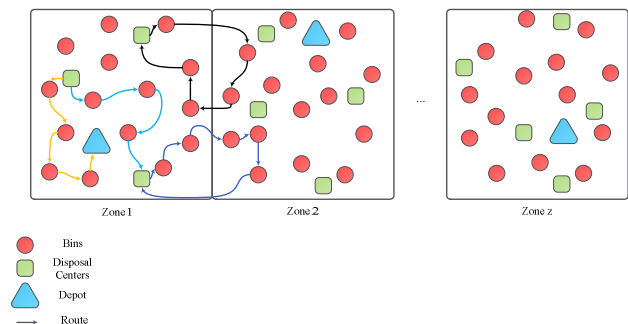


Figure 1: Map of the city

- Each vehicle that is located in a zone can collect wastes from the bins located in other zones, but it cannot unload wastes at a disposal center which is in another zone.
- Every vehicle works consecutively while there are no wastes in any vehicle.
- Vehicles load wastes from the bins until there is no more capacity, then it departs to a disposal center, unload, starts another trip and collects waste from the bins that are not empty.
- The number of vehicles of a specific kind available at each depot is unrestricted[11].
- The collection operation is mechanized and be performed just by vehicles. So, only necessary human resource is the driver.

Model notations Sets

- D_i Set of depots in i th zone; $\{1\}$
- F_i set of bins in i th zone; $\{2, \dots, 2 + f_i\}$
- B_i set of bins in i th zone; $\{3 + f_i, \dots, 2 + f_i + b_i\}$
- N_i Set of all nodes in i th zone (depots, disposal centers and bins); $\{D_i, F_i, B_i\} = \{1, 2, \dots, 1 + f_i + b_i\}$
- Z Set of zones.
- K Set of type of vehicles.
- M_{ij} Set of vehicles of type i in zone j .

Parameters

- l_{ipk} Loading and unloading duration for vehicle of type k at point i in zone p (for all $i \in N_p, p \in Z, k \in K$).
- w_{ip} Waste volume exist at point i in zone p (for all $i \in B_p, p \in Z$).
- T_{ipkjq} Traveling time from point i in zone p to point j in zone q by vehicle of type k (for all $i \in N_p, j \in N_q, p, q \in Z, k \in K$).
- v_k Capacity of vehicle type k .

- FC_k Fixed cost of vehicle type k .
- VC_k Variable cost of vehicle type k .
- d_i Number of depots in zone i .
- f_i Number of disposal centers in zone i .
- b_i number of bins in zone i .
- H A very big value.

Variables

- $r(i, j, p, q)$ 1, if point i in zone p is connected to point j in zone q ; 0, otherwise (for all $i \in N_p, j \in N_q, p, q \in Z$).
- $c(i, p, m, k, z)$ 1, if m th vehicle of type k in zone z allocates to point i in zone p ; 0, otherwise (for all $i \in N_p, m \in M_{kz}, k \in K, p, z \in Z$).
- q_{ipk} Waste volume exist in vehicle of type k after leaving point i in zone p (for all $i \in N_p, p \in Z, k \in K$).

Model Formulation

Waste collection routing problem that is proposed in this paper can be formulated as a mixed integer programming model. Proposed model formulation can be represented as following equations:

$$\begin{aligned}
 & \text{minimize } \sum_{k \in K} \left(FC_k \left(\sum_{z \in Z} \sum_{m \in M_{kz}} \sum_{p \in Z} \sum_{i \in D_p} c(i, p, m, k, z) \right) + VC_k \left(\sum_{q \in Z} \sum_{j \in N_q} \sum_{p \in Z} \sum_{i \in N_p} (r(i, j, p, q) (T_{ipkjq} + l_{ipk})) \right) \right) + \\
 & \left| \sum_{q \in Z} \sum_{j \in F_q} \sum_{p \in Z} \sum_{i \in N_p} r(i, j, p, q) - \frac{\sum_{q \in Z} \sum_{j \in F_q} \sum_{p \in Z} \sum_{i \in N_p} r(i, j, p, q)}{\sum_{s=1}^Z f_s} \right| \tag{1}
 \end{aligned}$$

Subject to :

$$c(i, p, m, k, z) - c(j, q, m, k, z) \leq 1 - r(i, j, p, q) \quad \forall i \in N_p, j \in N_q, p, q, z \in Z, k \in K, m \in M_{kz}, \tag{2}$$

$$c(i, p, m, k, z) - c(j, q, m, k, z) \geq r(i, j, p, q) - 1 \quad \forall i \in N_p, j \in N_q, p, q, z \in Z, k \in K, m \in M_{kz}, \tag{3}$$

$$c(i, p, m, k, z) \leq 1 \quad \forall i \in D_p, p, z \in Z, k \in K, m \in M_{kz}, \tag{4}$$

$$q_{ipk} = 0 \quad \forall i \in F_p, D_p; k \in K; p \in Z, \quad (5)$$

$$q_{ipk} - H(1 - c(i, p, m, k, z)) \leq V_k \quad \forall i \in B_p; p, z \in Z; k \in K; m \in M_{kz}, \quad (6)$$

$$\sum_{z \in Z} \sum_{k \in K} \sum_{m \in M_{kz}} c(i, p, m, k, z) = 1 \quad \forall i \in B_p; p \in Z, \quad (7)$$

$$\sum_{p \in Z} \sum_{i \in B_p, D_p} r(i, j, p, q) = 1 \quad \forall j \in B_q; q \in Z, \quad (8)$$

$$\sum_{q \in Z} \sum_{j \in B_p, D_p} r(i, j, p, q) = 1 \quad \forall i \in B_p; p \in Z, \quad (9)$$

$$q_{jqk} - w_{jq} - q_{ipk} \geq -H(1 - r(i, j, p, q)c(j, p, m, k, z)) \quad \forall i \in N_p; j \in B_q; p, q \in Z, \quad (10)$$

$$q_{jqk} - w_{jq} - q_{ipk} \leq H(1 - r(i, j, p, q)c(j, p, m, k, z)) \quad \forall i \in N_p; j \in B_q; p, q \in Z, \quad (11)$$

$$c(i, p, m, k, z) = 0 \quad \forall i \in N_p \setminus B_p; p, z \in Z; p \neq z; m \in M_{kz}; k \in K, \quad (12)$$

$$r(i, j, p, q) = 0 \quad \forall i \in B_p; j \in D_q; p, q \in Z, \quad (13)$$

$$r(i, j, p, q) = 0 \quad \forall i \in D_p; j \in F_q, D_q; p, q \in Z, \quad (14)$$

$$r(i, j, p, q) = 0 \quad \forall i \in F_p; j \in F_q, D_q; p, q \in Z, \quad (15)$$

$$c(i, p, m, k, z), r(i, j, p, q) = \{0, 1\} \quad \forall i \in N_p; j \in N_q; p, q, z \in Z; k \in K; m \in M_{kz}, \quad (16)$$

$$q_{ipk} \geq 0 \quad i \in N_p; p \in Z; k \in K, \quad (17)$$

The objective function (1) attempt to minimize the total operation cost of vehicles including fixed and variable costs and also minimizes the number of vehicles are used. The last part of the objective function balances unloading at disposal centers[13]. In fact this part is the absolute difference between the total number of unloading at each disposal center and the average number of unloading at all of the disposal centers. Constraint (2) and (3) enforce each two points which are connected must be serviced by the same vehicle. On the other hand this constraint relates r and c variables together. Constraint (4) ensures that every vehicle must be used for only one route. Equation (5) guarantees that vehicle which allocates to a disposal center must unload all of exist wastes and on the other hand, vehicle after leaving depots is empty of wastes. Constraint (6) ensures that volume of waste load by each vehicle does not

exceed from the capacity of vehicle. Constraint (7) ensures that just one vehicle allocate to each bin. Constraint (8) guarantees that just one vehicle from other bins goes to node j and Constraint (9) ensures that just one flow exit from bin i . On the other hand if it is available to load waste by a vehicle it must load all off the waste. Constraints (10) and (11) calculate the waste volume exist in a vehicle after leaving a point. Constraint (12) guarantees any vehicle from a zone does not go to the disposal center which located in another zone. Equation (13) prohibits that vehicles go to depots directly from bins. Constraint (14) guarantees that any empty vehicle does not go to disposal centers and prohibits vehicle goes to a depot from another, immediately. Equation (15) ensures that any vehicle does not go to a disposal center directly from another. Constraints (16) and (17) specify the ranges of variables.

Experimental Results

In this section, for showing applicability of the proposed model, a small examples including random parameters' value are presented. In this example, two types of vehicles are considered that have 1000000 and 2000000 units as vehicles fixed cost, 900 and 2000 units as vehicle variable cost and 5 and 14 units as vehicles capacity for waste collection, respectively. But, one of each type of vehicles is considered. 8 nodes are considered including depots, disposal centers and bins. For instance, table (1)

shows the traveling times between nodes according to vehicle type. Also, table (2) shows the loading times for each node and vehicle type. Represented model solved by Lingo software and the experimental results are presented in table (3) and (4). In table (3) 1 means there is relationship between those nodes and 0 means not. In table (4), 1 means that vehicle is allocated to that node and 0 means not. Global Optimum objective function's value earned 3277301 units.

Table 1. Traveling time between nodes.

Traveling time matrix	Vehicle type	Node number								
		1	2	3	4	5	6	7	8	
Node number	1	1	0	36	30	96	30	48	78	96
		2	0	22.5	18.75	60	18.75	30	48.75	60
	2	1	36	0	18	60	42	24	42	54
		2	22.5	0	11.25	37.5	26.25	15	26.25	33.75
	3	1	30	18	0	66	54	42	48	66
		2	18.75	11.25	0	41.25	33.75	26.25	30	41.25
	4	1	96	60	66	0	90	72	72	66
		2	60	37.5	41.25	0	56.25	45	45	41.25
	5	1	30	42	54	90	0	24	48	72
		2	18.75	26.25	33.75	56.25	0	15	30	45
	6	1	48	24	42	72	24	0	24	48
		2	30	15	26.25	45	15	0	15	30
	7	1	78	42	48	72	48	24	0	54
		2	48.75	26.25	30	45	30	15	0	33.75
	8	1	96	54	66	66	72	48	54	0
		2	60	33.75	41.25	41.25	45	30	33.75	0

Table 2. Loading time for each number and vehicle type.

Loading time	Vehicle type		
	1	2	
Node Number	1	0	0
	2	8	15
	3	1	0.6
	4	5	2
	5	0	0
	6	8	4
	7	2	1
	8	3	1

Table 3. Relation between nodes.

Relations between nodes		Node number							
		1	2	3	4	5	6	7	8
Node number	1	0	0	1	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	3	0	1	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	1
	5	0	0	0	0	0	0	1	0
	6	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	1	0	0
	8	0	0	0	1	0	0	0	0

Table 4. Vehicle allocation to nodes.

Vehicle allocation		Vehicle type	
		1	2
Node number	1	1	0
	2	1	0
	3	1	0
	4	1	0
	5	0	1
	6	0	1
	7	0	1
	8	1	0

Conclusion and Future Research Directions

In this study a waste collection problem is investigated. Proposed model is a multi-zone waste collection routing problem with intermediate facilities. In model’s network three types of nodes are considered. Waste bins, disposal centers as intermediate facilities and depots. Each zone can have just one depot. Vehicles that perform waste collection are considered heterogeneous that have different cost, speed and capacity that are located in depots in each zone. Vehicles can start a trip from a disposal center to non-empty waste bins and after exceeding vehicle capacity return to disposal centers and unload collected wastes. Vehicles can load wastes from every zone of the city but they should unload them in a disposal center which belong to that zone. In order to model the described problem mathematically, a mixed integer programming is proposed in this paper. Mathematical model attempt to minimize the total operation cost of vehicles including fixed and variable costs and also minimizes the number of vehicles are used. Last term of objective function is the absolute difference between the total number of unloading at each disposal center and the average number of unloading at all of the disposal centers that balances unloading at disposal centers. Finally presented model solve for a small size problem using random generated parameters’ value to show applicability of the proposed mathematical model.

Because of high level importance of waste collection in urban management, future research directions can concentrate on application of proposed model to special cases with special conditions. Also, for large size problems that can be faced in actual cases, heuristic and metaheuristic algorithms such as tabu search etc. can be utilized. In actual environment some parameters of this problem such as amount of waste in bins are not known, so amount of waste can be considered as a stochastic parameter. In this regard stochastic models can be utilized to solve the problem.

References

- [1] S. Alumur, B.Y. Kara, “A new model for the hazardous waste location-routing problem”, *Computers & Operations Research*,34, 1406–1423, 2007.
- [2] S. K. Amponsah, S. Salhi, “The investigation of a class of capacitated arc routing problems: the collection of garbage in developing countries”, *Waste Management*, to appear, 2004.
- [3] R. Aringhiera, M. Bruglieri, F. Malucelli, M. Nonato, “An asymmetric vehicle routing problem arising in the collection and disposal of special waste”, *Electronic Notes in Discrete Mathematics*, 17, 41–47, 2004.
- [4] S. Baptista, R. C. Oliveira and E. Zúquete, “A period vehicle routing case study”,

- European Journal of Operational Research, 139, 220–229, 2002.
- [5] A.M. Benjamin, J.E. Beasley, “Metaheuristics for the waste collection vehicle routing problem with time windows, driver rest period and multiple disposal facilities”, *Computers & Operations Research*, 37, 2270–2280, 2010.
- [6] D. Bommisetty, M. Dessouky, L. Jacobs, “Scheduling collection of recyclable material at Northern Illinois University campus using a two-phase algorithm”, *Computers & Industrial Engineering*, 35, 435–438, 1998.
- [7] B. Crevier, J. Cordeau, G. Laporte, “The multi-depot vehicle routing problem with inter-depot routes”, *European journal of operational research*, 176, 2, 756–773, 2007.
- [8] H.A. Eiselt, M. Gendreau, G. Laporte, “Arc routing problems, part 1: the Chinese postman problem”, *Operations Research*, 43, 3, 399–414, 1995.
- [9] G. Ghiani, F. Guerriero, G. Laporte, R. Musmanno, “Tabu Search Heuristics for the Arc Routing Problem with Intermediate Facilities under Capacity and Length Restrictions”, *Journal of Mathematical Modelling and Algorithms*, 3, 209–223, 2004.
- [10] V. Hemmelmayr, K. F. Doerner, R. F. Hartl, S. Rath, “A heuristic solution method for node routing based solid waste collection problems”, *J Heuristics*, 2011, DOI 10.1007/s10732-011-9188-9.
- [11] S. Irnich, “A multi-depot pickup and delivery problem with a single hub and heterogeneous vehicles”, *European Journal of Operational Research*, 12, 310–328, 2000.
- [12] J. Kytöjoki, T. Nuortio, O. Bräysy, M. Gendreau, “An efficient variable neighborhood search heuristic for very large scale vehicle routing problems”, *computers and operations research*, 34, 9, 2743–2757, 2007.
- [13] J.Q. Li, D. Borenstein, P.B. Mirchandani, “Truck scheduling for solid waste collection in the City of Porto Alegre, Brazil”, *Omega*, 36, 1133 – 1149, 2008.
- [14] M.C. Mouäro, M.T. Almeida, “Lower-bounding and heuristic methods for a refuse collection vehicle routing problem”, *European Journal of Operational Research*, 121, 420–434, 2000.
- [15] T. Nuortio, J. Kytöjoki, H. Niska, O. Bräysy, “Improved route planning and scheduling of waste collection and transport”, *Expert Systems with Applications*, 30, 223–232, 2006.
- [16] H.L. Ong, T.N. Goh, K.L. Poh, “A computerized vehicle routing system for refuse collection”, *Advances in Engineering Software*, 12, 54–58, 1990.
- [17] M. Polacek, K.F. Doerner, R.F. Hartl, V. Maniezzo, “A variable neighborhood search for the capacitated arc routing problem with intermediate facilities”, *Journal of Heuristics*, 14, 405–423, 2008.
- [18] J. Teixeira, A. Antunes, J. de Sousa, “Recyclable waste collection planning, a case study”, *European Journal of Operational Research*, 158, 3, 543–554, 2004.